647510 **BSCbooCC5030** **Seat No :_________**

# B.Sc. Semester - 5 (*CBCS*) Examination
# Oct/Nov.– 2019
# BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1(CORE)

**Time: 2:30 Hours** **Marks: 70**

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate marks.

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**1 (A) Answer any two:** **(10)**

(1) Define: Equivalence relation. If A is nonempty set and R is an equivalence relation defined in A then prove that

(i) $b \in [a] \Longrightarrow [a] = [b]$

(ii) $[a] = [b] \Leftrightarrow aRb$

(iii) either $[a] = [b]$ or $[a] \cap [b] = \emptyset$.

(2) Define: Lattice as an algebraic structure. If $(L, \leq)$ is a lattice and $a, b \in L$ then prove that

$(i) a \leq b \Leftrightarrow a \vee b = b \Leftrightarrow a \wedge b = a$

$(ii) b \leq c \Longrightarrow a \wedge b \leq a \wedge c$ and $a \vee b \leq a \vee c$

(3) Define: Direct product of two lattice and prove that direct product of two lattices is also lattice.

**(B) Answer any one:** **(04)**

(1) Define: Lattice as a poset and show that $(S_{30}, D)$ is a lattice.

(2) If $L_1$ is the lattice $D_6 = \{1,2,3,6\}$ (divisors of 6) and $L_2$ is the lattice $(P(S), \subseteq)$ where $S = \{a, b\}$ then show that $D_6$ is isomorphic to $P(S)$.

**2 (A) Answer any two:** **(10)**

(1) If $(B, *, \oplus, ', 0, 1)$ is Boolean algebra then $\forall x_1, x_2 \in B$, prove that:

(i) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$ (ii) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$

(2) State and prove D’ Morgan’s law for Boolean algebra.

(3) State and prove Stone’s representation theorem for Boolean algebra.

**(B) Answer any one:** **(04)**

(1) Define: Atoms in Boolean algebra and find all atoms of $(s_{30}, *, , ', 0, 1)$.

(2) Express $\alpha(x_1, x_2, x_3) = x_1 \oplus x_2$ as the sum of product canonical form.

**3 (A) Answer any two:** **(10)**

(1) State and prove Cauchy-Riemann conditions in Cartesian form for analytic function.

(2) If $f(z) = u + iv$ is an analytic function then in usual notation prove that:

$$\nabla^2 |f(z)|^2 = 4 \left| f'(z) \right|^2.$$

(3) Prove that $u = r^2 sin2\theta$ is harmonic function and find its conjugate function.

**(B) Answer any one:** **(04)**

(1) Find an analytic function $f(z)$ such that $R\left[f'(z)\right] = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$.

(2) Define: Entire function and show that $z^2$ is entire function.

**4 (A) Answer any two:** **(10)**

(1) State and prove Cauchy’s integral formula.

(2) If C is the circle $|z| = R, R > 1$ described in the counter clock direction then show that $\left| \int_C \frac{logz}{z^2} \right| < 2\pi \left( \frac{\pi + logR}{R} \right)$

(3) Evaluate: $\int_{c} f(z)dz$, where $f(z) = \frac{z+2}{2}$

where c is (i) The semi circle $z = 2e^{i\theta}, 0 \leq \theta \leq \pi$

(ii)The semi circle $z = 2e^{i\theta}, \pi \leq \theta \leq 2\pi$

**(B) Answer any one:** **(04)**

(1) Prove that $\int_{c} \pi e^{\pi \acute{z}}\, dz = 4(e^{\pi} - 1)$, where c is the square joining points $z = 0, z = 1, z = 1 + i \wedge z = i$.

(2) Define: (i) Continuous arc (ii) Closed curve. Evaluate

$$\int_{i}^{i+1} (x - y + ix^2)dz$$

**5 (A) Answer any two:** **(10)**

(1) State and prove Liouville's theorem.

(2) Using Cauchy's integral formula for derivatives deduce that

$$f^n(z_0) = \frac{n!}{2\pi i}\int \frac{f(z)}{(z - z_0)^{n+1}}dz$$

(3) Prove that $\int_{C} \frac{e^{kz}}{z} dz = 2\pi i, C: |z| = 1$ and hence deduce that

$$\int_{0}^{\pi} e^{kcos\theta} cos(ksin\theta)d\theta = \pi.$$

**(B) Answer any one:** **(04)**

(1) If $g(z_0) = \int_{C} \frac{4z^2+z+5}{z-z_0} dz, c: |z| = 2$ then find $g(1), g(-1), g(i), g^{'}(-i), g^{'}(1)$.

(2) Evaluate: $\int_{c} \frac{sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz$, where $c: |z| = 1$.

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